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Mark Scheme (Results)

Summer 2013

GCE Mechanics 4 (6680/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Rules for Marking Mechanics

- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is accuracy error not method error.

- Omission of mass from a resolution is method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *ONCE* per complete question.

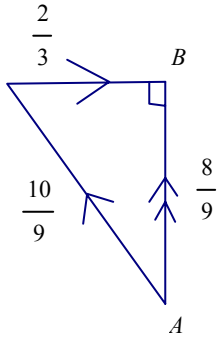
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

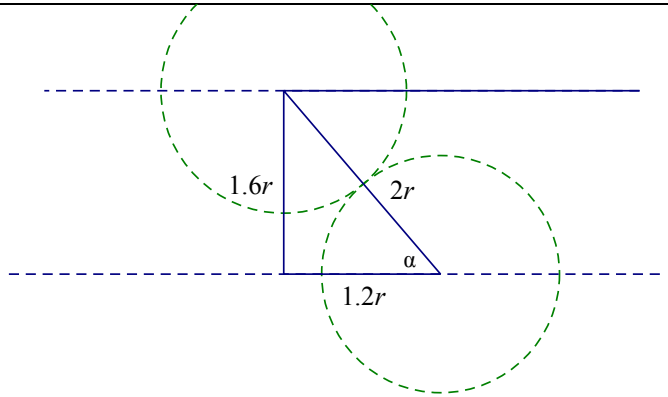
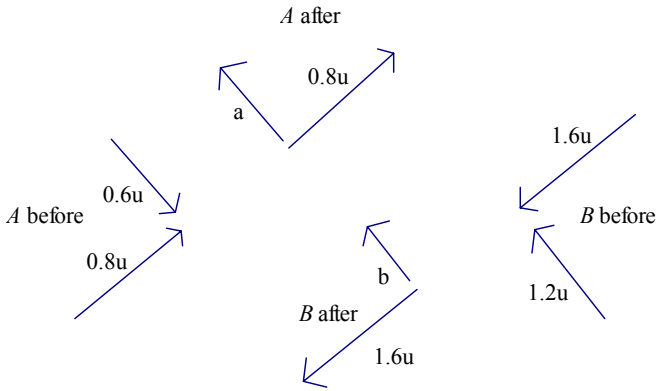
- Accept column vectors in all cases.

Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.

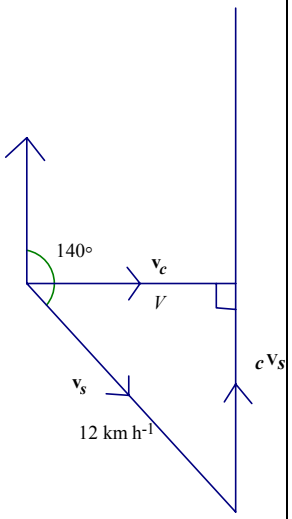
Question Number	Scheme	Marks	
1(a)	Equation of motion: $\frac{1}{2}g-\frac{3}{2}v=\frac{1}{2}\frac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1	Differential equation. All 3 terms required but condone sign errors
	NB: these two marks are available in (b) if not scored in (a)		
	$\int 1\mathrm{d}t=\int\frac{1}{9.8-3v}\mathrm{d}v$	M1	Separate the variables and attempt to integrate
	$t+(C)=-\frac{1}{3}\ln(9.8-3v)$	A1=A1	A1 for each side. C not needed
	$t=0,\ v=0\Rightarrow C=-\frac{1}{3}\ln 9.8$	M1	Use initial conditions to evaluate Cor limits on a definite integral.
	$t=-\frac{1}{3}\ln\left(\frac{9.8-3v}{9.8}\right)$	A1	Or equivalent
	$3v=9.8\left(1-e^{-3t}\right)$ ***Given Answer***	A1 (8)	**Watch out.** cwo
(a) alt	Equation of motion: $\frac{1}{2}g-\frac{3}{2}v=\frac{1}{2}\frac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1	All 3 terms required but condone sign errors
	$e^{3t}\frac{\mathrm{d}v}{\mathrm{d}t}+3e^{3t}v=ge^{3t}$, $\frac{\mathrm{d}}{\mathrm{d}t}\left(ve^{3t}\right)=ge^{3t}$	M1	Use of integrating factor e^{3t}
	$ve^{3t}=\frac{1}{3}ge^{3t}(+c)$	A1=A1	A1 for each side. $+C$ not required.
	$t=0, v=0\Rightarrow 0=\frac{1}{3}g+C$	M1	Use initial conditions to evaluate C
	$\Rightarrow ve^{3t}=\frac{1}{3}g\left(e^{3t}-1\right),\ 3v=9.8\left(1-e^{-3t}\right)$	A1 A1	Correct equation in any equivalent form **Given form** cwo

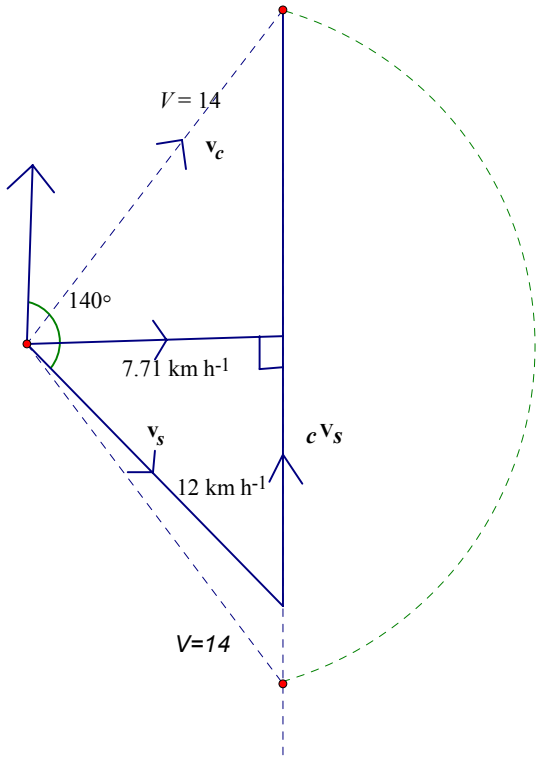
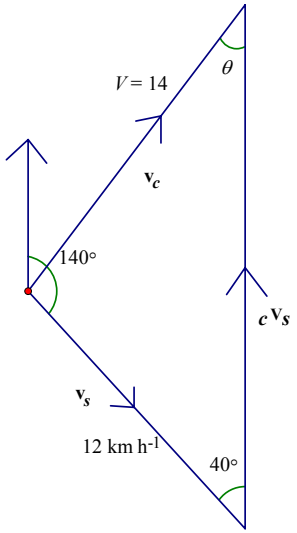
Question Number	Scheme	Marks	
1(b)	$\frac{dx}{dt} = \frac{9.8}{3}\left(1 - e^{-3t}\right) \Rightarrow x = \frac{9.8}{3}\left(t + \frac{1}{3}e^{-3t}\right)(+C)$ $t = 0, x = 0 \Rightarrow C = -\frac{9.8}{9}$ $t = 2, x \approx 5.4$ (m)	M1 A1 M1 A1 A1 (5) **(13)**	Integrate the given v to find x C not needed Use the initial conditions to evaluate C or use limits correctly in a definite integral 5.45, $\frac{g}{9}\left(5 + e^{-6}\right)$ or equivalent
(b) alt	$g - 3v = v\frac{dv}{dx}$		
	$\int 1dx = \int \frac{v}{g - 3v}dv = \int -\frac{1}{3} + \frac{g}{3(g - 3v)}dv$	M1	Separate the variables and rearrange the RHS
	$x = -\frac{v}{3} - \frac{g}{9}\ln(g - 3v) + C$	A1	$+C$ not needed
	$x = 0, v = 0 \Rightarrow C = \frac{g}{9}\ln g$ and $t = 2, v = \frac{g}{3}\left(1 - e^{-6}\right)(= 3.258....)$	M1 A1	Use the initial conditions to find C & find the value of v when $t = 2$
	$x = \frac{g}{9}\left(1 - e^{-6}\right) - \frac{g}{9}\ln\left(e^{-6}\right) = 5.4$	A1	
		(5)	
		(13)	

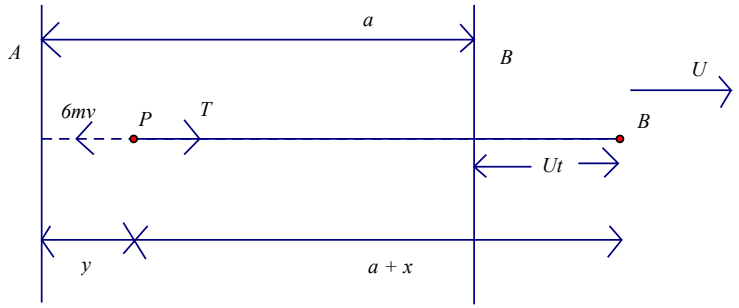
Question Number	Scheme	Marks	
2(a)	Shortest time $50 \div \frac{10}{9}$, $= 45$ (s)	M1,A1	
(b)	Drifts $\frac{2}{3} \times "45"$, $= 30$ (m)	M1 A1	$\frac{2}{3} \times$ their time
(c)	 Trig or pythag to find velocity of swimmer in direction AB $\frac{8}{9}$ ms^{-1} $50 \div "\frac{8}{9}"$, $= 56.25$ (s)	M1 A1 DM1,A1 **(8)**	0.88 or better Dependent on the previous M 56 or better

Question Number	Scheme	Marks	
3			
	$0.6u$ or $u\cos\alpha$	B1	component of the initial velocity of A parallel to the line of centres on impact
	$1.2u$ or $2u\cos\alpha$	B1	component of the initial velocity of B parallel to the line of centres on impact
	$2m\times1.2u-3m\times0.6u=3ma+2mb$	M1	CLM parallel to the line of centres. Requires all the terms.
	$(3a+2b=0.6u)$	A1ft	Correct unsimplified for their $0.6u$ and $1.2u$
	$e(1.2u+0.6u)=a-b$	M1	Restitution parallel to the line of centres. Must be used the right way round.
	$(a-b=0.3u)$	A1ft	Correct unsimplified for their $0.6u$ and $1.2u$ If signs are inconsistent between the two equations, penalise here.
		DM1	Solve a pair of simultaneous eqns in a & b for one of a & b. Dependent on the two previous M marks.
	$a=0.24u$ or $b=-0.06u$	A1	In terms of u only
	$(1.2u-(-0.06u))\times2m=2.52mu$	M1	Find impulse on A or B. Unsimplified. For their a or b. Correct mass for the velocities used.
	or $(0.24u-(-0.6u))\times3m=2.52mu$	A1 **(10)**	$\frac{63}{25}$

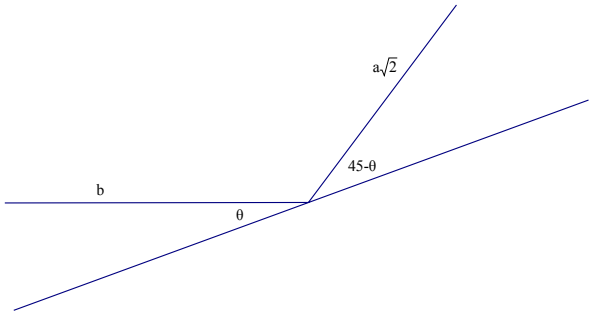
Question Number	Scheme	Marks	
4(a)	PE of ring $= -mgx$	B1	Taking the level of the peg as zero PE
	PE of particle $= -3mg(L-\sqrt{x^2+d^2})$	M1 A1	
	$\Rightarrow V = 3mg\sqrt{x^2+d^2} - mgx + \text{constant}$. **AG**	A1 (4)	Watch out
(b)	$\dfrac{\mathrm{d}V}{\mathrm{d}x} = \dfrac{3mg.2x}{2\sqrt{x^2+d^2}} - mg$	M1	
	$\dfrac{dV}{dx} = 0 \Rightarrow 3x = \sqrt{x^2+d^2}\ , 9x^2 = x^2+d^2,\ 8x^2 = d^2$	M1	Set $\dfrac{dV}{dx} = 0$ and solve for x
	$x = \dfrac{d}{\sqrt{8}} = \left(\dfrac{\sqrt{2}d}{4}\right)$	A1	$0.354d$ of better
(c)	$\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2} = 3mg\left(\dfrac{\sqrt{x^2+d^2}.1 - x.\dfrac{2x}{2\sqrt{x^2+d^2}}}{x^2+d^2}\right) =$	M1	Product or quotient rule $\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2} = \dfrac{3mg}{\sqrt{x^2+d^2}} - \dfrac{3mgx}{2}.2x.\left(x^2+d^2\right)^{-\frac{3}{2}}$
	$3mg\left(\dfrac{\sqrt{9x^2}.1 - x.\dfrac{2x}{2\sqrt{9x^2}}}{9x^2}\right) = \dfrac{3mgd^2}{\left(x^2+d^2\right)^{\frac{3}{2}}}\ (>0)$	A1	OR $= 3mg\left(\dfrac{3x - \dfrac{x}{3}}{9x^2}\right)(>0)$ Correct unsimplified.
			$\dfrac{16\sqrt{2}mg}{9d},\ 2.5\dfrac{mg}{d},\ \dfrac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = \dfrac{9mgd}{\sqrt{8}}$
	Stable	A1ft **(10)**	Correct conclusion for their expression

Question Number	Scheme	Marks	
5(a)	 Minimum $V = 12\cos 50°$ ≈ 7.71	M1 A1 A1	Use of triangle with right angle between v_C and ${}_C v_S$. Condone sin/cos confusion. Correct unsimplified trig expression 7.71 only

Question Number	Scheme	Marks	
5(b)			
	Vector triangle for relative velocities when $V = 14$	M1	could have relative velocity due S. Could show both possibilities.
	Select the vector triangle with the relative velocity due N.	A1	
	$\frac{\sin\theta}{12} = \frac{\sin 40}{14}$	DM1 A1	Use of sine rule or equivalent to find θ
	Bearing 033°	A1 **(8)**	Final answer. Accept 33.4°

Question Number	Scheme	Marks	
6(a)			
	$a+Ut=y+(a+x)$	M1	Diagram or clear explanation using distances
	$Ut=x+y$ ***Answer Given***	A1	Watch out for fudges.
(b)	$T=\dfrac{9ma\times x}{a}=9mx$	B1	
	$T-6m\dot{y}=m\ddot{y}$	M1	Equation of motion of P. Requires all 3 terms in terms of x and/or y
	$9mx-6m(U-\dot{x})=-m\ddot{x}$	A2	Expressed in terms of x. -1 each error
	$\ddot{x}+6\dot{x}+9x=6U$	A1	**Answer given**. Watch out for fudges
(c)	$t=0, x=0, \dot{x}=U \qquad 0=AU+\dfrac{2U}{3},\ A=-\dfrac{2}{3}$	M1 A1	Use initial conditions to find A
	$\dot{x}=BUe^{-3t}-3\left(A+Bt\right)Ue^{-3t}$	M1 A1	Differentiate
	$U=BU-3AU,\ B=3A+1=-1$	A1	
(d)	$\dot{y}=U-\dot{x}=U-\left(-Ue^{-3t}+2Ue^{-3t}+3Ute^{-3t}\right)$	M1	
	$=U\left(1-e^{-3t}-3te^{-3t}\right)$	A1	Or equivalent
		(14)	

Question Number	Scheme	Marks	
7(a)	State that impulse acts perpendicular to the wall and demonstrate that $(2\mathbf{i} + \mathbf{j}).(-\mathbf{i} + 2\mathbf{j}) = 0$	B1	Requires scalar product or gradient diagram.
(b)	Impulse momentum equation: $m(\mathbf{v} - \mathbf{u}) = m[(a-b)\mathbf{i} + a\mathbf{j}] = \lambda(-\mathbf{i} + 2\mathbf{j})$ $\Rightarrow a = -2(a-b)\,,\ 3a = 2b$	M1 A2 A1	Requires all terms present and of the correct structure -1 each error
	OR Taking scalar products of velocities with $(2\mathbf{i} + \mathbf{j})$ $\begin{pmatrix} b \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2b$ and $\begin{pmatrix} a \\ a \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 3a$ No change parallel to the wall so $2b = 3a$.	M1 A1A1 A1	
	Scalar products with $(-\mathbf{i} + 2\mathbf{j})$: $\begin{pmatrix} b \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix} = -b$ and $\begin{pmatrix} a \\ a \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix} = a$ Impact equation: $a = eb$ $e = \frac{2}{3}$	B1 M1A1 A1	

Question Number	Scheme	Marks	
7(b) alt			
	$b\cos\theta = a\sqrt{2}\cos(45-\theta)$	M1	Parallel to the wall. Condone trig confusion?
	$b\cos\theta = a\cos\theta + a\sin\theta$, $2b-2a=a$	A2	-1 each error. Both angles in same variable?
	$2b=3a$	A1	
	Use of $\tan\theta = \frac{1}{2}$	B1	When seen in (b). Implied by 26.6 or 18.4
	$a\sqrt{2}\sin(45-\theta) = eb\sin\theta$	M1	Perpendicular to the wall. Condone consistent trig confusion?
	$a\cos\theta = (a+eb)\sin\theta$, $2a=a+eb$	A1	$e=\sqrt{\frac{10a^2}{b^2}-4}$
	$e=\frac{2}{3}$	A1	0.67 or better
(c)	Fraction of KE lost $= \frac{b^2-2a^2}{b^2}$	M1A1	
	$= \frac{1-2\times\frac{4}{9}}{1} = \frac{1}{9}$	A1	
		(12)	

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